

# Learning Partial Models for Planning

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# Motivation

- Partial models
  - Compact size
  - Focused (limited) knowledge about the system dynamics
  - Easier to learn

# Outline

- Planning framework
  - Partial model
  - State-variable hierarchy
  - Partial model planning (PMP)
- Learning partial models
- Empirical evaluation

# Planning Framework

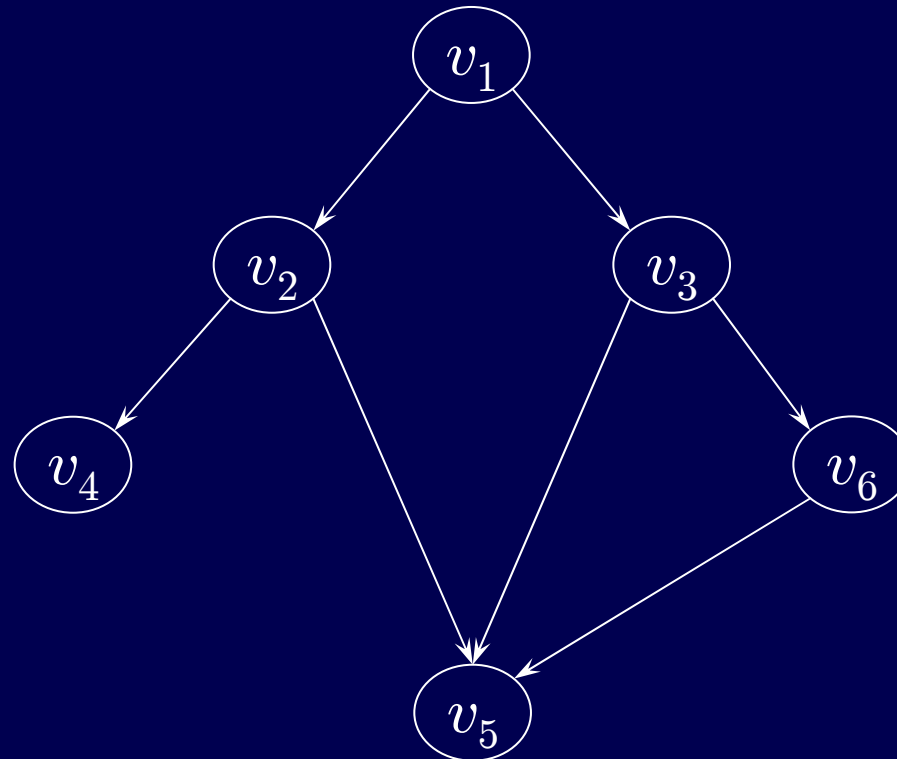
- Deterministic factored state space
  - $S = \text{Total state space} = D(v_1) \times \dots \times D(v_n)$
- Partial action model
  - Set of rules =  $\{r = op : pre \rightarrow post\}$
  - Relevant variables  $\rho(r) = \text{variables that are checked}$   
= variables that are changed
  - Example
    - $r = \text{North}: * 0 * \rightarrow * 1 *$
    - $\rho(r) = \{v_2\}$

# Partial Model

- Given ordering  $\Omega$  on variables ( $i < j \Leftrightarrow v_i \prec v_j$ )
  - Primary variable  $\psi(r, \Omega) = \text{first variable in } \rho(r)$
  - Secondary variables =  $\rho(r) - \psi(r, \Omega)$
  - $r.op$  has no effect on any variable preceding  $\psi(r, \Omega)$
- Projection of a rule  $r(v) = (\text{pre}[v], \text{post}[v])$
- Component graph (CG) for a variable
$$G_v = (D(v), \{r(v) : v = \psi(r, \Omega)\})$$
- Partial model (PM) =  $\{(G_v, R_v)\}$

# Hierarchy of variables

- The ordering is a “hierarchy” over variables



# Definitions

- A partial model is **adequate** iff there exists an ordering such that all CGs are strongly connected
- A domain is **serializable** iff there exists an ordering such every variable can be set to its desired value without affecting precursors

# Statement of Partial Model Planning

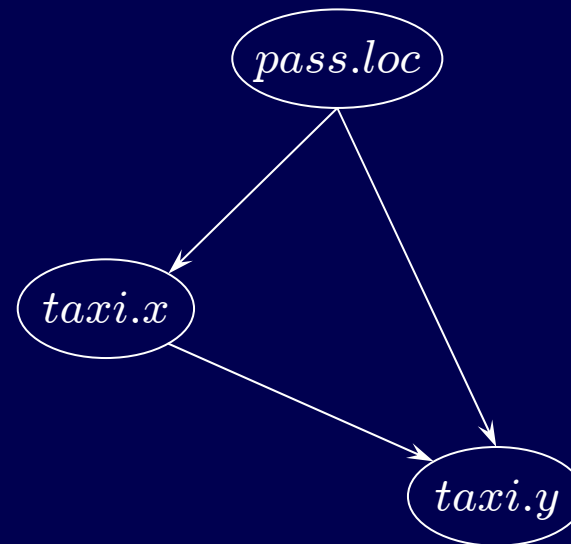
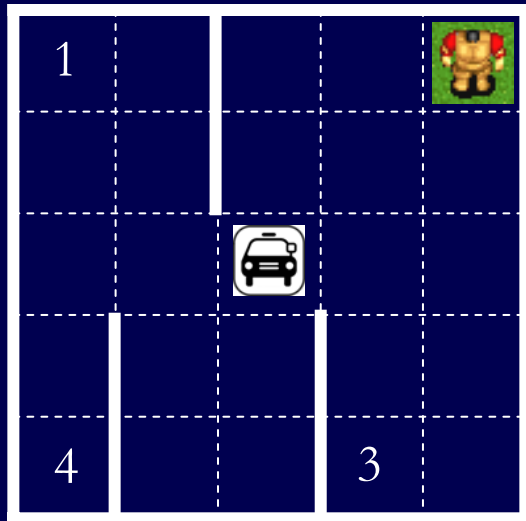
- If a domain has an adequate partial model then
  - $S$  is strongly connected via the operators in the PM
  - The domain is serializable
- This follows because every CG
  - Is strongly connected
  - Only affects the associated variable and successors



# Taxi Domain

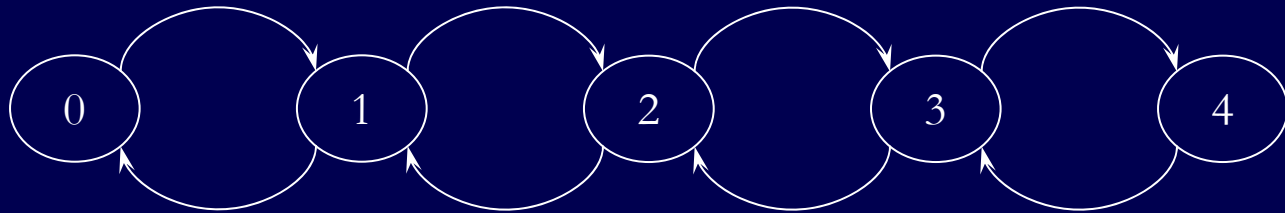
Actions:

- North, South, East, West
- Pickup
- Dropoff

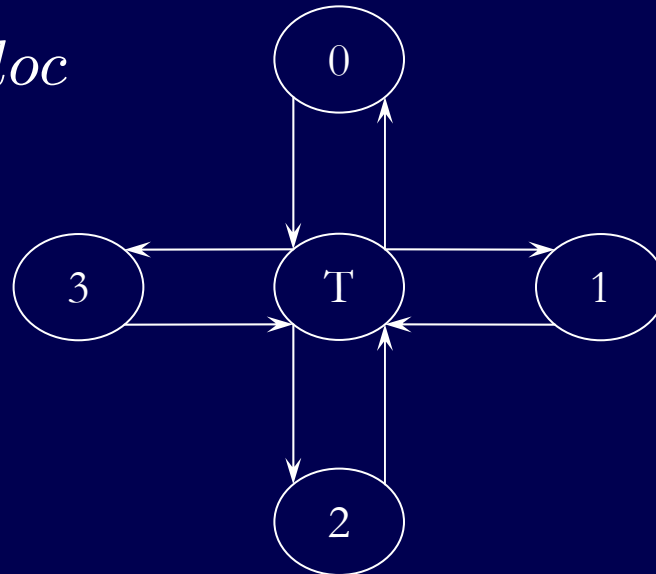


# Taxi Domain CGs

*taxi.x, taxi.y*

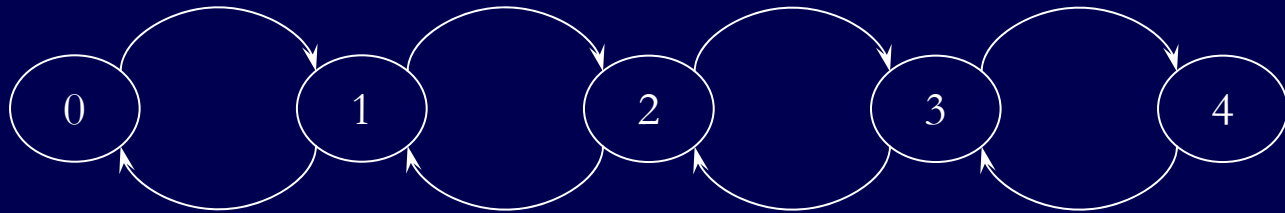


*pass.loc*



# Taxi Domain CGs

*taxi.x*



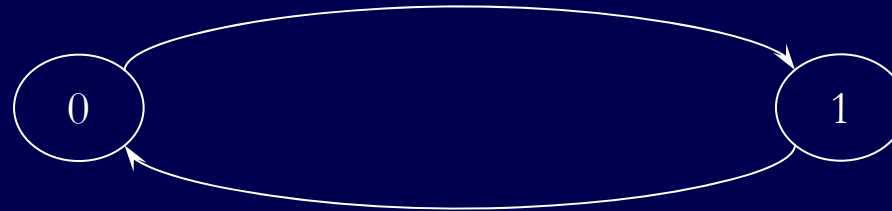
# Taxi Domain CGs

*taxi.x*

East:  $0\ 2^* \rightarrow 1\ 2^*$

East:  $0\ 3^* \rightarrow 1\ 3^*$

East:  $0\ 4^* \rightarrow 1\ 4^*$



West:  $1\ 2^* \rightarrow 0\ 2^*$

West:  $1\ 3^* \rightarrow 0\ 3^*$

West:  $1\ 4^* \rightarrow 0\ 4^*$

# Partial Model Planning

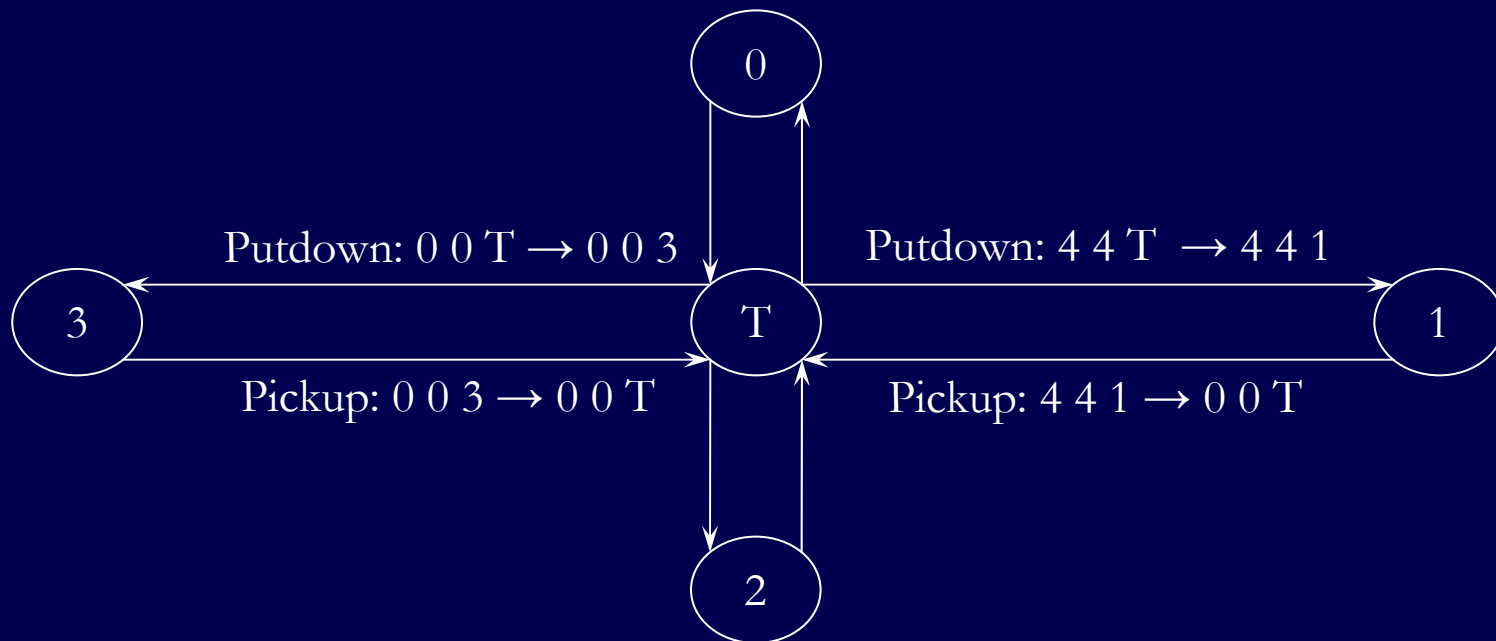
Start state: 3 0 3

Goal state: \* \* 1

# Partial Model Planning

Start state: 3 0 3

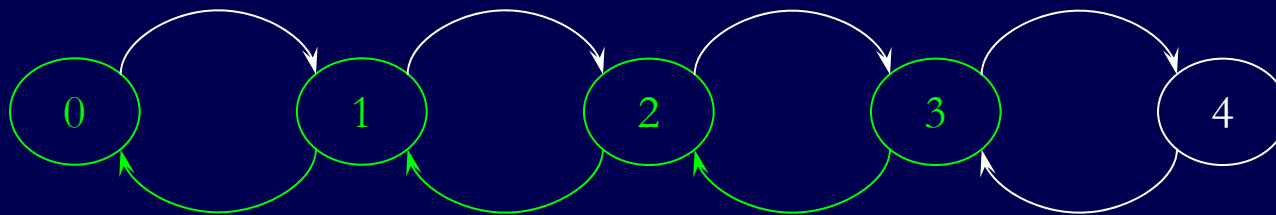
Goal state: \* \* 1



# Partial Model Planning

Start state: 3 0 \*

Goal state: 0 \* \*

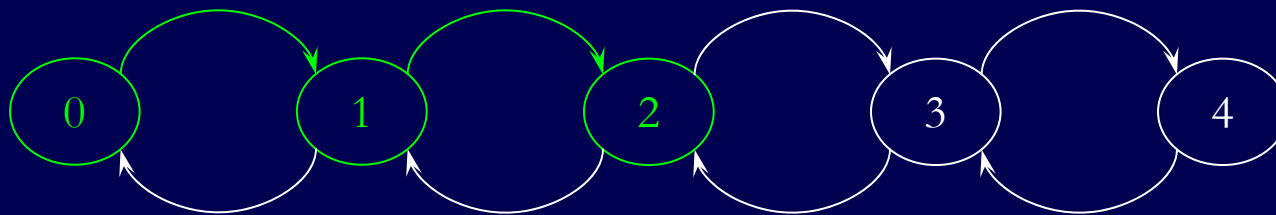


*taxi.x*

# Planning Algorithm

Start state: \* 0 \*

Goal state: \* 2 \*



*taxi.y*

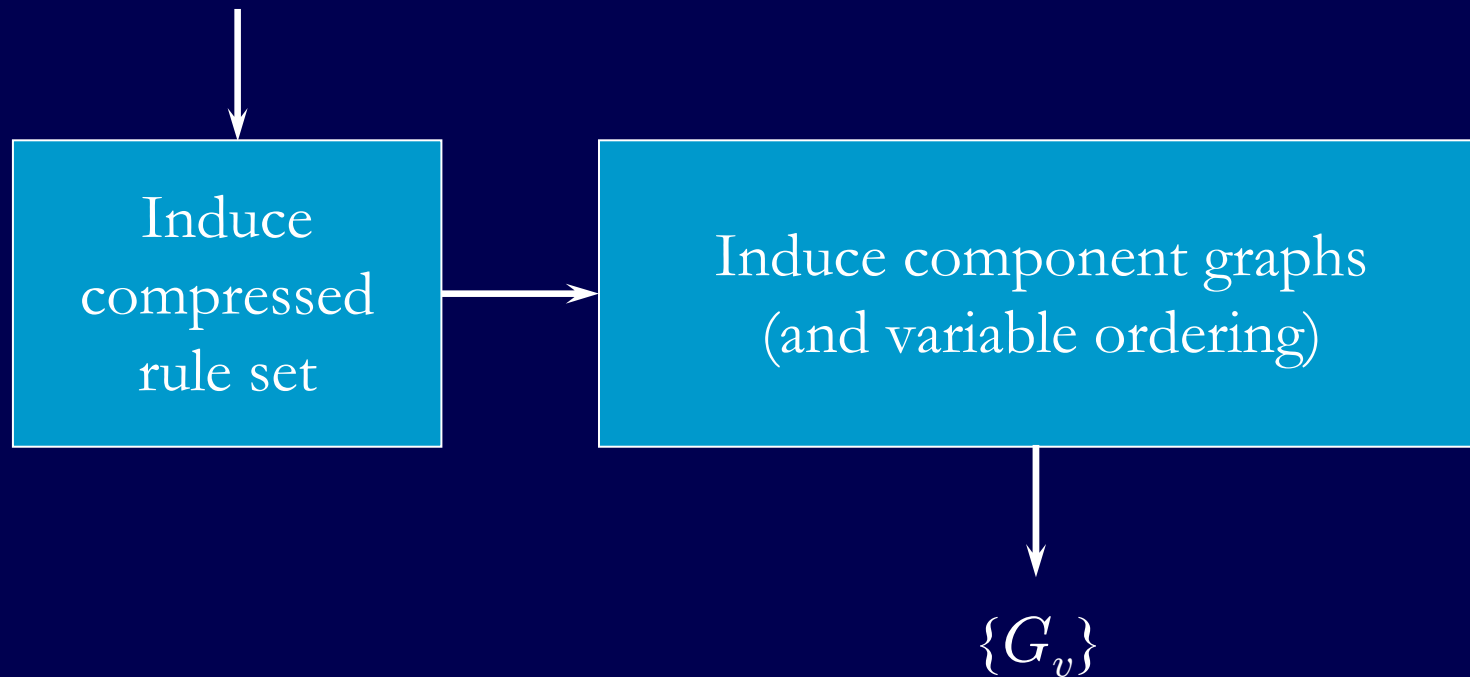


# Notions Of Optimality

- Parameters:
  - $D = \max_v D(v)$
  - $b = \max_r |\rho(r)| - 1$
  - $d =$  depth of variable hierarchy
  - $k =$  max number of rules on an edge
- Global optimality:  $O(D^n)$
- Hierarchical optimality:  $O((Dbk)^n)$ 
  - Pick best rule to minimize pre and post cost
- Recursive optimality:  $O((Dbk)^d)$ 
  - Pick best rule to minimize pre cost
- Agnostic(?) optimality:  $O((Db)^d)$ 
  - Pick an arbitrary rule on the shortest path
  - Assumes all pre costs are equivalent

# Learning Partial Models

$$T = \{(s, a, r, s')\}$$



# Inducing Compressed Rule Set

- $R \leftarrow \emptyset$
- While  $T$ 
  - Pick a transition  $(s, a, r, s')$  such that  $s \neq s'$
  - Let  $r = a : s \rightarrow s'$
  - Initialize  $\rho = \{v : r.pre[v] \neq r.post[v]\}$
  - $\rho = \text{LearnContext}(\{(s, a, r, s')\}, r, \rho)$
  - For  $v \notin \rho$ ,  $r.pre[v] \leftarrow r.post[v] \leftarrow *$
  - $R \leftarrow R \cup \{r\}$
  - $T \leftarrow T - \{(s, a, r, s') \text{ consistent with } r\}$

# Inducing Compressed Rule Set

0 2 2

East

1 2 2

West

0 2 2

South

0 1 2

West

0 1 2

# Inducing Compressed Rule Set

0 2 2

East

1 2 2

West

0 2 2

South

0 1 2

West

0 1 2

$r = \text{East: } 0^{**} \rightarrow 1^{**}$

# Inducing Compressed Rule Set

0 2 2

East

1 2 2

West

0 2 2

South

0 1 2

East

0 1 2

$r = \text{East: } 0^{**} \rightarrow 1^{**}$

# Inducing Compressed Rule Set

0 2 2

East

1 2 2

West

0 2 2

South

0 1 2

East

0 1 2

$$r = \text{East: } 0 2^* \rightarrow 1 2^*$$

# Inducing Compressed Rule Set

1 2 2

West

0 2 2

South

0 1 2

East

0 1 2

$r = \text{East: } 0 2^* \rightarrow 1 2^*$



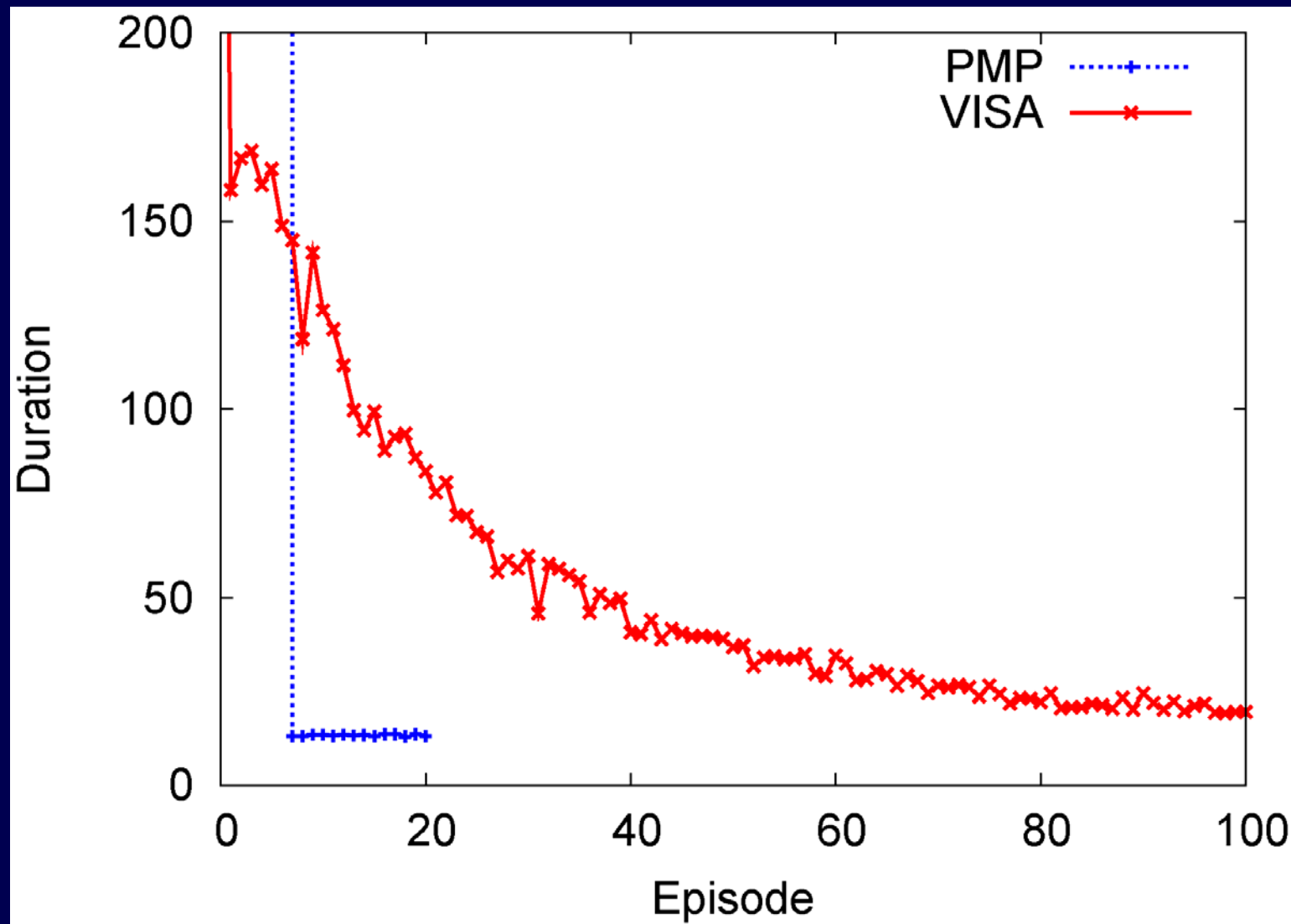
# Inducing Component Graphs

- While  $\Omega$  does not contain all variables
  - For  $r \in R$ 
    - $v \in \rho(r) - \{u : u \text{ in } \Omega\}$  // Assuming a single variable
    - Insert edge  $(r.pre[v], r.post[v])$  in  $G_v$
  - For  $v : v \notin \Omega \wedge G_v$ 
    - If  $G_v$  is strongly connected, append  $v$  to  $\Omega$
  - Delete all malformed component graphs

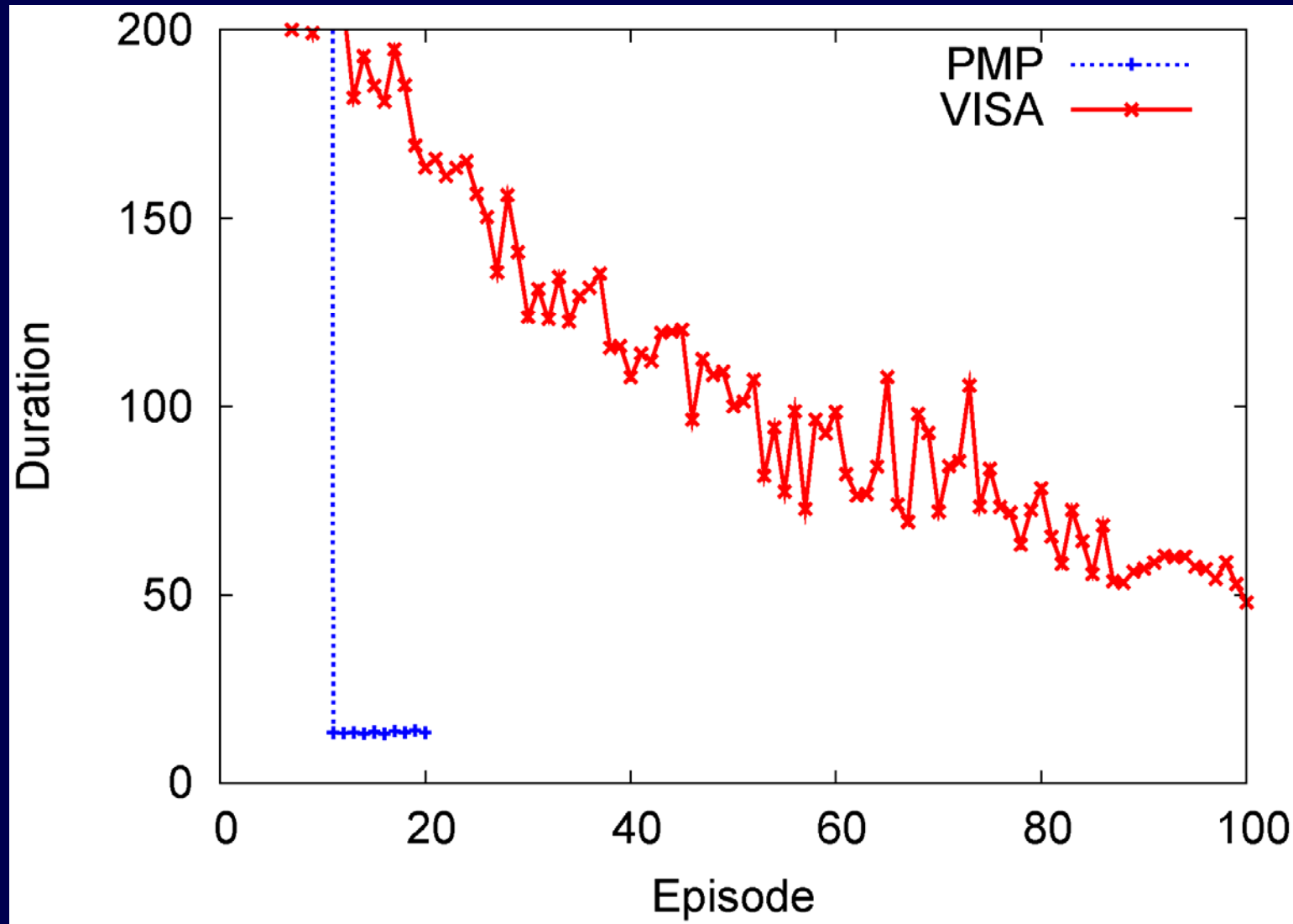
# Empirical Evaluation

- Compare against VISA (Jonsson & Barto, 06)
  - Learns an exit-option hierarchy from fully specified DBN models
  - Does intra-option Q learning
- Domains
  - Regular Taxi domain
  - Modified Taxi domain
    - Pickup when  $pass.loc = T$  causes  $taxi.y++$

# Empirical Evaluation: Regular Taxi



# Empirical Evaluation: Modified Taxi



# Results

- PMP learns the appropriate variable ordering and an adequate model in  $\sim 8$  episodes
- Recursive optimality coincides with global optimality in the Taxi domain
- VISA induces tight structure in Regular Taxi but only the shallowest hierarchy for Modified Taxi
  - Entire causal graph is strongly connected
- PMP's partial model stays unchanged

# Conclusion

- Partial models can be learned from random trajectories
- PMP can be made exponential in  $d$  even when VISA and factored planning are exponential in  $n$
- Future work
  - Bounding PMP's plan length vs. optimal
  - Generalizing factors to sets of variables
  - Easing the requirement of strong connectedness